Exam Seat No:_______________ Enrollment No:__________________

C.U.SHAH UNIVERSITY

WADHWAN CITY

University (Winter) Examination -2013

Course Name :M.Sc(Maths) Sem-I **Subject Name: -Differential Equations** **Marks :70**

Duration :- 3:00 Hours **Date : 27/12/2013**

Instructions:-

(1) Attempt all Questions of both sections in same answer book / Supplementary.
(2) Use of Programmable calculator & any other electronic instrument is prohibited.
(3) Instructions written on main answer Book are strictly to be obeyed.
(4) Draw neat diagrams & figures (If necessary) at right places.
(5) Assume suitable & Perfect data if needed.

SECTION-I

Q-1 a) Determine the radius of convergence of e^x. (02)

b) Evaluate: $\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right)\,\Gamma\left(\frac{5}{2}\right)$. (01)

c) Are $\sin x$ and $\cos x$ linearly independent? (01)

d) Write generating function of Bessel's function. (01)

e) Find n such that $\int_{-1}^{1} P_n(x)\ dx = 2$. (01)

f) Write Legendre's equation. (01)

Q-2 a) Find the series solution about $x = 0$ for $y'' + y = 0$. (05)

b) Show that $x = \infty$ is a regular singular point of $x^2y'' + 4xy' + 2y = 0$. (05)

c) Solve the differential equation $y'' + y = \sec x$ by the method of variation parameters. (04)

OR

Q-2 a) Find the series solution about $x = 0$ for $y' - 2xy = 0$. (05)

b) Determine the radii of convergence of the following series. (05)

$i)\ \sum_{n=1}^{\infty} \frac{n!}{n^n}\ x^n,\ (ii)\ \sum_{n=0}^{\infty} n^2x^n$

c) Determine the singular points of differential equation $2x\,(x-2)^2y'' + 3xy' + (x-2)y = 0$ and classify them as regular or irregular. (04)

Q-3 a) Find the series solution about $x = 0$ for $2x^2y'' - 3xy' + (3-x)y = 0$. (07)

b) By using Rodrigues formula, find $P_n(x)$, where $n = 0, 1,2,3,4$. (07)

OR

Q-3 a) Show that $\int_{-1}^{1} P_n(x)P_m(x)\ dx = \begin{cases} 0, & m \neq n \\ \frac{2}{2n+1}, & m = n \end{cases}$. Also evaluate $\int_{-1}^{1} P_3(x)P_2(x)\ dx$. (07)

b) Prove that: $i)\ J_{\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi\, x}}\sin x,\quad ii)\ J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi\, x}}\cos x,$ $iii)\ \frac{d}{dx}\ J_0(x) = -J_1(x)$ (07)

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SECTION-II

Q-4 a) Eliminate the constants a and b from $z = (x + a)(y + b)$. (02)

b) Solve: $z = px + qy + \sqrt{1 + p^2 + q^2}$. (01)

c) Solve: $xp + yq = z$. (01)

d) Under which condition Pfaffian differential equation in three variables is integrable. (01)

e) Write Lagrange's equation. (01)

f) Define: Hypergeometric function. (01)

Q-5 a) Find integral curve of the simultaneous differential equation (05)

$$\frac{dx}{zx} = \frac{dy}{-zy} = \frac{dz}{y^2 - x^2}.$$

b) Using Picard's method of successive approximations, find the third approximation of the solution of equation: $\frac{dy}{dx} = x + y^2$, where $y = 0$ when $x = 0$. (05)

c) Eliminate arbitrary function f from $z = xy + f(x^2 + y^2)$. (04)

OR

Q-5 a) Find integral curve of the simultaneous differential equation (05)

$$\frac{dx}{x^2 + y^2} = \frac{dy}{2xy} = \frac{dz}{(x + y)z}.$$

b) Solve $(y^2 + z^2)dz + xy\,dy + xzdz = 0$ by using Natani's method. (05)

c) Eliminate arbitrary function f from $z = f\left(\frac{xy}{z}\right)$. (04)

Q-6 a) A necessary and sufficient condition that there exists between two functions $u(x, y)$ and $v(x, y)$ a relation $F(u, v) = 0$ not involving x or y explicitly is that $\frac{\partial(u,v)}{\partial(x,y)} = 0$. (07)

b) Find complete integral of $2(z + xp + yq) = yp^2$ by using Charpit's method. (07)

OR

Q-6 a) Show that a complete integral of $f(u_x, u_y, u_z) = 0$ is $u = ax + by + cz + d$ where $f(a, b, c) = 0$. Also find the complete integral of $u_x + u_y + u_z - u_x u_y u_z = 0$. (07)

b) Prove: (07)

$(i)\ F(-n, 1; 1; -x) = (1 + x)^n$,

$(ii)\ F'(a, b; c; x) = \frac{ab}{c} F(a + 1, b + 1; c + 1; x)$.

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